Massless Particles Orbiting a (2+1) dimensional charged black hole

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Abstract:

The purpose of this poster is to present orbits of photons around a charged black hole. The black hole considered is a 2+1 dimensional black hole with cosmological constant. The black hole has mass M and charge Q. A thorough analysis of the geometry around the black hole and the horizons will be presented. The effective potential, will also be discussed in relative to the mass and the charge of the black hole.

Introduction

A black hole is a region of space that contains an object that is so massive and dense that even light cannot escape. The distance from the center and to the no return radius is referred to as the event horizon. The distance where light stays in a circular orbit around the black hole is known as the photon sphere. There are only three observable properties of black holes: mass, electric charge and angular momentum, all other properties about any material inside the black hole will be lost; there will be nothing left of its former composition, structure, or history. We can also study the gravitational field of a black hole by looking at the motions of particles around a black hole. In this poster, we will model the motions of massless particles orbiting a (2 + 1) dimensional charged black hole.

Effective Potential and Energy

The total energy for a neutral particle can be expressed as:

$$E^2 = \dot{r}^2 + V(r) \qquad (1)$$

Where $\dot{r}$ represents the time derivative of the distance, and V(r) represents the potential:

$$V(r) = \left(\frac{L^2}{r^2} + \epsilon^2 \right) f(r) \qquad (2)$$

Where $\epsilon^2 = 0$

L represents the angular momentum, and ϵ^2 represents the type of particle that we are dealing with

$\epsilon^2 = 1$ for massive
$\epsilon^2 = 0$ for massless

Mathematically, we can define the geometry, f(r), of the black hole as the following :

$$f(r) = -M - \Lambda r^2 + \frac{4\,q}{3r} \qquad (3)$$

Where, $\Lambda = -1$

M represents the mass of the black hole, q represents the charge, Λ represents the cosmological constant, and r represents the distance from the center. The event horizon, r_h, is found by setting this equation equal to zero. If we set L equal to 3, and q equal to 3, the potential becomes:

$$V_e = \frac{9(-M + 4r + r^2)}{r^2} \qquad (4)$$

Motion of Particles

We can look at different effective potentials, by changing the mass of the black hole. As given in Fig.1.

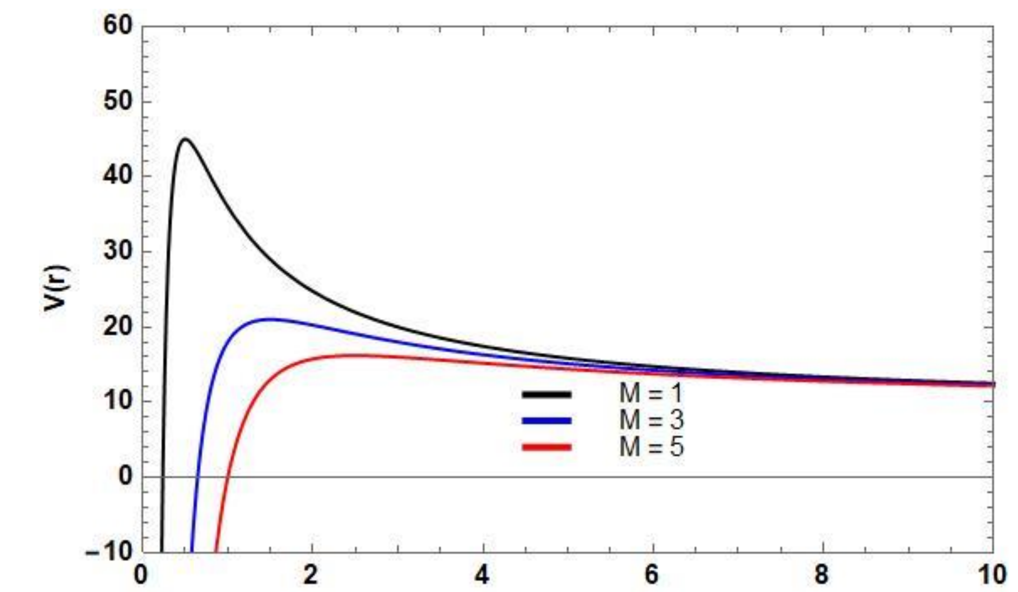

Figure 1: The Potential of the black hole, plotted for M values

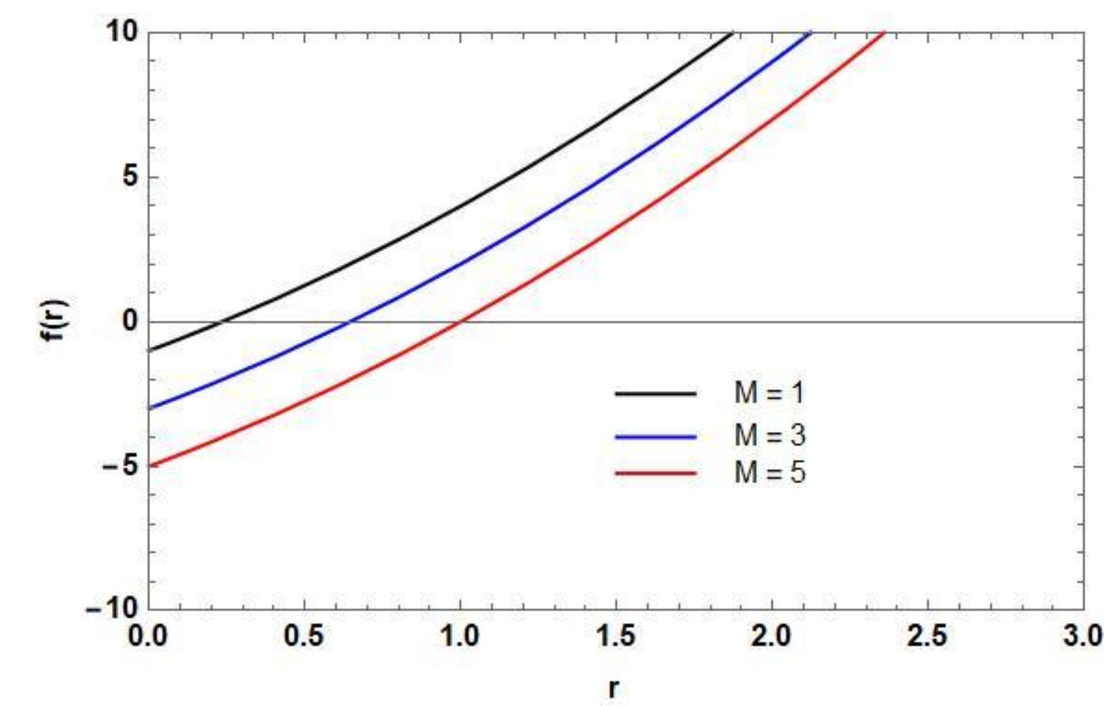

Figure 2: The geometry of the black hole, plotted for different M values

Photon Sphere

Equation 1 allows us to relate the particles motion to the and effective potential and total energy (Where $E^2 \geq V_e$)

$$E_n^2 - V_e = \dot{r}^2 \qquad (5)$$

We can use this relation to determine the distance where the motion of the particle remains constant, or when

$$\dot{r}^2 = 0$$

$$E^2 - V_e = 0$$

Which can only occur when $E_n^2 = V_e$. or

$$\frac{d(V_e)}{dr} = 0 \qquad (6)$$

$$\frac{9(4+2r)}{r^2} - \frac{18(-M+4r+r^2)}{r^3} = 0$$

Solving for r:

$$r_c = \frac{M}{2} \qquad (7)$$

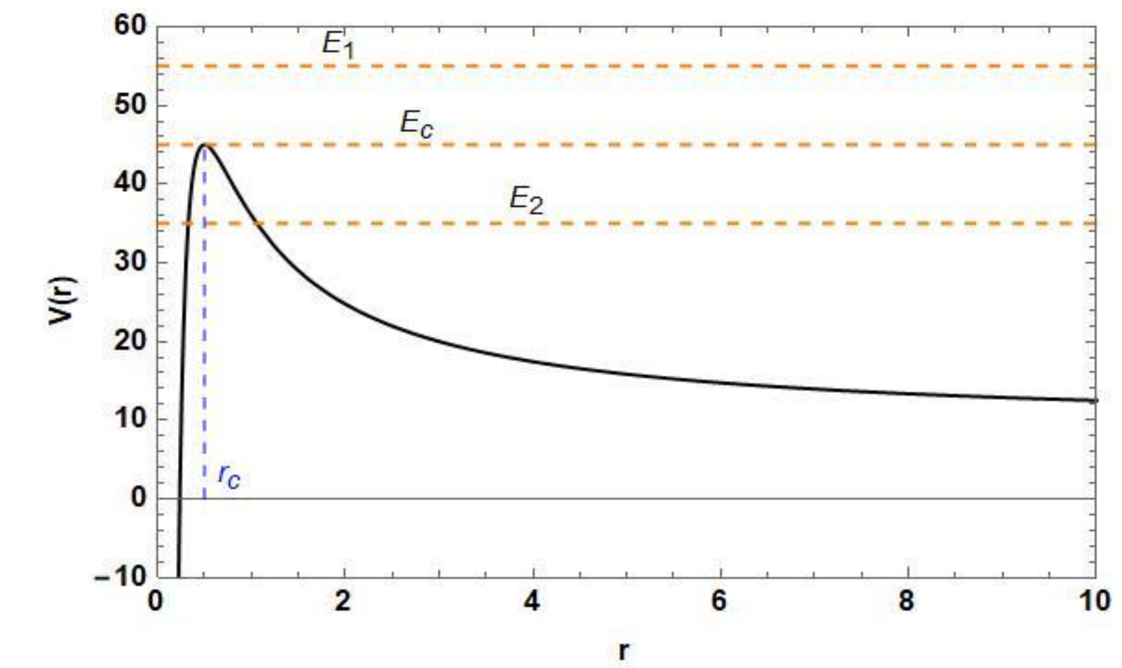

Figure 3: The potential of the black hole, plotted with different values of E (where M = 1).

So, $r_c = \frac{1}{2}$

If $\mathbf{E_n = E_c}$:

As expected the photons will exhibit circular motion around the black hole at a distance of r_c, if the total energy of the photons are equal to that of the effective potential.

If $E_n = E_1$:

At this energy level, $E_1^2 - V_e > 0$, therefore $\dot{r}^2 > 0$, which means that the photons will fall directly into the singularity.

If $E_n = E_2$:

At values greater than r_c the photons will fly out to greater distances away from the singularity; at values less than r_c, the photons will fall into the singularity.

Conclusion

The plots of how photons move around a black hole for the (2+1) dimensional black hole show that there is a strong relationship between the radius of the photon sphere and mass of the black hole . Furthermore, there appears to be an even greater relationship between the effective potential and mass, leading to less variation for what could happen if the photons energy levels were below that of E_c. Overall, the plots of the model shown in this poster agree with similar models shown in previous work (Fernando 2012).

Future Work

We will plot exact trajectories of the photons. We will also study charged particles around the same black hole.

References

Fernando, S., Schwarzschild black hole surrounded by quintessence: null geodesics, 44, General Relativity and Gravitation | article, D: 10.1007/s10714-012-1368-x, (2012) .

Cataldo, M., Cruz, N., del Campo, S., et al. (2000) , (/2+1)-dimensional black hole with Coulomb-like field, 484, Physics Letters B | article, D: 10.1016/S0370-2693(00)00609-2